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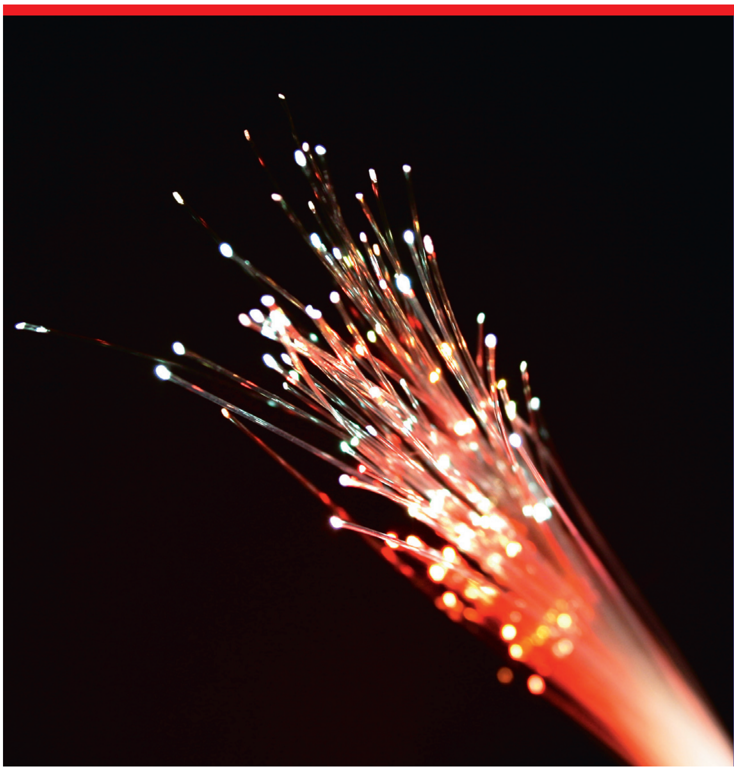

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A Survey of the Routing and Wavelength Assignment Problem

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Abstract

In an all-optical network, optical fibers are used to transmit data. An optical fiber carries light along its length at high rates and with little loss. Several wavelengths on a single fiber can be used to transfer data, when using wavelength-division multiplexing. In this way, several data transmissions at very high speed can take place on a single fiber.

When transmitting data in an all-optical network, data connections must be established in such a way, that two or more connections never share a wavelength on the same fiber. The routing and wavelength assignment (RWA) problem consists of finding a path and a wavelength for a set of data connections. The objective is typically to maximize the profit of established data connections or to minimize the cost of establishing all data connections. The RWA is $\mathcal{NP}$-hard, thus much research has been conducted to finding a good way of approaching the RWA.

This paper introduces the RWA and lists a number of restrictions from the literature on the RWA and on the underlying network topology. An overview of heuristic, metaheuristic and exact solution methods is given. Running times for the heuristic methods are presented and computational results from the literature are discussed.

1 Introduction

The use of optical fibers in telecommunication infrastructure is ever increasing. An optical fiber carries light along its length at very high rates and with little loss. When data is sent via an optical fiber, it is transmitted on a certain wavelength of light. A fiber can carry several independent transmissions, each by a different wavelength. The *wavelength-division multiplexing* (WDM) technology allows multiple optical carrier signals on a single optical fiber. WDM works on a circuit switched network, i.e., in a network where the connection between nodes and terminals is established before use, and where the wavelength is not shared with other traffic. For a technical overview of optical fibers, see Halsall [29], and for more information on the WDM, see Thiele and Nebeling [38] or the thesis of Jue [39].

The problem of finding a good way of establishing data connections and of assigning wavelengths to the different connections, is denoted the *routing and wavelength assignment* (RWA) problem. Two or more data connections are not allowed to share the same wavelength on the same edge. Constraints can be set on whether or not wavelengths can be converted. If wavelength conversion is possible, then further constraints can be set on where conversion may take place, and on the range of wavelengths, into which a wavelength can be converted.

The RWA problem can be considered as a *static* problem, where wavelengths of every future connection are reserved all at once. Another viewpoint is the *dynamic* version of RWA, where a wavelength is not reserved before it is needed, and where the wavelength is released when the corresponding data connection is no longer needed. The objective is typically to maximize the number of established connections or to minimize the number of used wavelengths.

The static RWA is applicable when customers have several data connection requests, and when the data connections are to be established permanently. The static RWA does not necessarily try to leave room for future connections, so it is mainly applicable when the current amount of data connection requests are also the only data connection requests.

The RWA is $\mathcal{NP}$-hard, thus several solution approaches are presented in the literature. A common approach is to decompose the RWA into two subproblems: the routing problem, and the wavelength assignment problem. The complexity of the routing problem depends on the chosen objective, while the wavelength assignment problem always is $\mathcal{NP}$-hard. Another approach is to solve the RWA problem as one problem. Methods for this include metaheuristics, and integer linear programming formulations. An overview of the proposed methods is presented in Table 1. The table shows what problem, each method works on, the complexity of each method, and finally gives references to the literature. Theoretical running times are only given for the constructive heuristics.

Some surveys on the RWA problem exist in the literature: Zang et al. [71] present a survey containing few routing approaches and many heuristics for the wavelength assignment problem. The latter are compared experimentally. Choi et al. [17] present a classification of existing methods for the RWA, where approaches are argued to be either search methods or selection methods. Furthermore, Choi et al. compare the performance of methods, but apart from a few theoretical running times, it is not clear what the comparisons are based on.

The contribution of this survey is the presentation of a much larger variety of solution methods than included in the surveys of Zang et al. and Choi et al. The presented methods include recently presented approaches from the literature. This paper does not only consider the decomposed RWA, but also presents metaheuristics and exact formulations of the overall RWA. Furthermore, experiments from the literature is gathered and discussed. No general benchmark instances are used in the literature, and the objective of solution methods differs. For these reasons, it is not trivial to decide which methods perform better, thus this survey also presents theoretical running times, and uses these along with test results in a performance analysis of the proposed solution methods. Finally, we give recommendations on future work in the RWA research area.

This survey is structured as follows. First, in Section 2, the RWA problem, and variants hereof are defined. The network topology is presented, i.e., constraints on whether or not wavelength conversion is allowed, etc. In Section 3, methods for solving the RWA problem heuristically are presented. These methods are all based on the decomposition of RWA into the two subproblems: the routing problem and the wavelength assignment problem. The section includes an overview of experimental results from the literature along with theoretical running times for the constructive heuristics. In Section 4 methods for the overall RWA is presented. These methods include metaheuristics, and integer linear programming formulations. The section contains experimental results from the literature. Concluding remarks are given in Section 5. This section includes conclusions on the performance analysis of the presented solution methods, and our recommendations on further work on the RWA.

Approach	**Problem**	**Sta./Dyn.**	**Complexity**	**Ref.**
`Fixed Routing`	Routing	Both	$\mathcal{O}(E + V \log V)$	[13, 17]
`Fixed-Alternate Routing`	Routing	Both	$\mathcal{O}(E + V \log V + k)$	[6, 13, 23]
`Adaptive Routing`	Routing	Dyn.	$\mathcal{O}(E + V \log V)$	[71]
`Least Congested Path Routing`	Routing	Both	$\mathcal{O}(E(E + V \log V))$	[14, 52]
`Shortest Path Adaptive Routing`	Routing	Both	$\mathcal{O}(E(E + V \log V))$	[52]
`Routing with Reduction of Wavelength Continuity Conflicts`	Routing	Both	Polynomial	[43, 42]
`Ant Colony Routing`	Routing	Sta.	Metaheuristic	[69]
`Genetic Algorithm`	Routing	Sta.	Metaheuristic	[6]
`Linear Programming`	Routing	Sta.	$\mathcal{NP}$-hard	[71]
`Graph Coloring`	WA	Both	$\mathcal{NP}$-hard	[71]
`Random Assignment`	WA	Both	$\mathcal{O}(WE)$	[67]
`First Fit Assignment`	WA	Both	$\mathcal{O}(WE)$	[47]
`Least Used Assignment`	WA	Both	$\mathcal{O}(W \log W + WE)$	[54, 71]
`Most Used Assignment`	WA	Both	$\mathcal{O}(W \log W + WE)$	[54, 71]
`Exhaustive Search Assignment`	WA	Both	$\mathcal{O}(WE)$	[54]
`Minimum Product Assignment`	WA	Both	$\mathcal{O}(WE)$	[37]
`Least Loaded Assignment`	WA	Both	$\mathcal{O}(WE)$	[40, 71]
`Maximum Sum Assignment`	WA	Both	$\mathcal{O}(kWE)$	[11, 67]
`Relative Capacity Loss Assignment`	WA	Both	$\mathcal{O}(kWE)$	[72]
`Distributed Relative Capacity Loss Heuristic`	WA	Both	$\mathcal{O}(kWE)$	[71]
`Wavelength Reservation Assignment`	WA	Dyn.	$\mathcal{O}(1)$	[13]
`Protecting Threshold Assignment`	WA	Dyn.	$\mathcal{O}(1)$	[13]
`Genetic Algorithm`	WA	Sta.	Metaheuristic	[30]
`Simulated Annealing`	WA	Sta.	Metaheuristic	[30]
`Tabu Search`	WA	Sta.	Metaheuristic	[30]
`Bin Packing Heuristic`	WA	Sta.	Metaheuristic	[65]
`Ant Colony Optimization`	RWA	Sta.	Metaheuristic	[4]
`Genetic Algorithm`	RWA	Sta.	Metaheuristic	[3, 64]
`Linear Programming`	RWA	Sta.	$\mathcal{NP}$-hard	[63]
`Linear Programming`	RWA	Sta.	$\mathcal{NP}$-hard	[71]
`Linear Programming`	RWA	Sta.	$\mathcal{NP}$-hard	[56]
`Linear Programming`	RWA	Sta.	$\mathcal{NP}$-hard	[34]
`Integer Multicommodity Flow Problem`	RWA	Sta.	$\mathcal{NP}$-hard	[12]
`Integer Programming`	RWA	Sta.	$\mathcal{NP}$-hard	[51]
`Integer Programming`	RWA	Sta.	$\mathcal{NP}$-hard	[35]

Table 1: An overview of all the methods, which are presented in this survey. The first column contains the name of the methods. Then follows problem types: the routing problem, the wavelength assignment problem (WA), or the RWA problem. The third column denotes whether or not, the method works on the static problem (Sta.) or the dynamic problem (Dyn.). The next column contains complexity: theoretic running times are only given for the heuristics. Finally, the right most column gives references to the literature for each method.

2 Problem Definition

In this section, details on the RWA and on the all-optical network are presented. First, we discuss common assumptions on the network in which to establish data connections. Next, the two main variants of the RWA, the static and the dynamic RWA, are further introduced.

2.1 Network Topology

The optical network is considered in an abstract manner. Technical details are omitted, instead we consider a *network* consisting of *nodes* and *edges*. Edges represent fiber links. An edge can hold several fibers, each potentially holding several wavelengths. Single-fiber is when each edge consists of only one fiber and multi-fiber is when each edge consists of several fibers. In this paper, we work on single-fiber networks unless else is mentioned. A node corresponds to any active equipment with an ingoing and/or outgoing edge. This could be a switch, a hub, an amplifier etc. A data connection request consists of a source and a target. A path with an assigned wavelength is to be found between the source and the target nodes. In the RWA, paths of different data connections are to be generated such that no two paths share the same edge and the same wavelength. That is, two paths using the same wavelength, must be edge disjoint. An example of the network representation is seen in Figure 1.

Figure 1: An example of a network representation of an optical network. Two data connections are routed through the network using the same wavelength. Thus, the two paths are edge disjoint.

When working on the RWA, some assumptions on *wavelength conversion* are made. A data connection may change wavelength when wavelength converters are available at intermediate nodes of the data connection path. In the literature, RWA works on different networks:

- There are no wavelength converters. In this case, a wavelength continuity constraint is imposed, see Zang et al. [71].
- Only a subset of nodes includes wavelength converters. This is denoted sparse wavelength conversion, see Iness and Mukherjee [32].
- All nodes include wavelength converters. The network is said to be wavelength convertible, see Ramamurthy and Mukherjee [62].

In the network representation, a switch with a wavelength converter attached is simply considered as one node. Comparisons of the different types of networks have been performed by Barry and Humblet [10], among others.

Furthermore, constraints on the usage of wavelength converters may be imposed. These constraints include sharing of converters, and limiting the range of possible conversions. Sharing converters may be beneficial. If converters are not shared, then the number of converters at a node increases. Lee and Li [50] have shown that when the number of wavelength converters at a node exceeds some threshold, then the performance of the network decreases.

Some converters only support changes of wavelengths within a certain range. E.g., the wavelength λ_i can be converted to wavelengths in the range $\lambda_{(i-k)}, \ldots, \lambda_i, \ldots, \lambda_{(i+k)}$, where k

is the range limitation factor. For more information on the limited-range wavelength converters, see the work of Iness and Mukherjee [32] or of Yates et al. [70].

When wavelength converters are only placed on certain nodes, much research has been conducted on network design, i.e., where to place the converters. Dutta and Rouskas [22] present a survey and a number of heuristics for the problem of designing the network. Koster and Zymolka [44, 45, 46] give lower bounds and then solve the problem of minimizing the number of required wavelength converters to optimality. A thorough analysis on the overall design of a WDM network is performed by Jue [39], and an analysis on how to place the components of an optical network is done by Iness [31].

2.2 Variants of the RWA

In the following, we consider both the static and the dynamic RWA. Recall that in the static RWA, all data connection requests are known in advance, they are to be established at the same time, and they are assumed to exist forever. An instance may hold more data connection requests than can be established; if a connection cannot be established, it is said to be *blocked*. Hence, the objective of the static RWA is typically to maximize either the number of established data connections or the profit of established data connections. The static RWA is proved to be $\mathcal{NP}$-hard by Chlamtac et al. [16]. The problem may be formulated mathematically as a mixed integer problem, see Ranaswami and Sivarajan [63].

In the dynamic RWA, data connection requests arrive with time; they are to be established at arrival time, and they are to be shut down at a given time. This means that wavelengths can be reused; when a data connection is shut down, its wavelength is released. As for the static case, blocking may occur. The objective of the dynamic RWA is typically to maximize the number of established data connections. Because no knowledge exists on future data connection requests, solutions to the dynamic RWA are local optimums.

The far majority of methods for solving the RWA apply to both the static and the dynamic RWA. In the following sections, solution methods from the literature are presented.

3 Decomposition of the RWA

Both the dynamic and the static RWA are difficult to solve. A reason for this is that the problems consist of two parts: routing data connections and assigning wavelength to data connections.

Both the static and the dynamic RWA are often solved by splitting the problem into two subproblems: the routing problem and the wavelength assignment problem, see e.g. Arteta et al. [4], Zang et al. [71] and Zheng and Mouftah [73]. First, routes for all connections are found. Next, wavelengths are assigned. The division of the problem makes it easier to solve, but solving the subproblems instead of the whole problem does not guarantee an optimal solution. Instead, dividing the RWA into two parts is a heuristic method.

Much research has been put into decomposing the RWA into these two parts. In this section, we present some of the routing algorithms and methods for wavelengths assignment from the literature.

3.1 Routing

The routing problem consists of finding a path between the source and the target of each data connection. The complexity of the routing problem depends on the objective. If we simply wish to connect a set of node pairs, then the problem can be solved polynomially using a shortest path algorithm. If the objective is to minimize the maximal number of paths on an edge, then the problem is $\mathcal{NP}$-hard, see e.g. Zang et al. [71].

Fixed Routing

The routing problem can be solved in polynomial time as an *all pairs shortest path* problem, see Ahuja et al. [2] for more information. This method is denoted `Fixed Routing`. The definition of *shortest* path varies; the length of a path may be measured in the number of used edges, or in the number of available bandwidths etc., see Birman and Kershenbaum [13] and Choi et al. [17]. In `Fixed Routing`, exactly one path is found per data connection.

Fixed-Alternate Routing

Another routing method is to find several paths between the pair of terminals for all data connections. If the paths for a data connection are edge disjoint, then the approach can be considered somewhat fault tolerant, i.e., if a connection fail on one path, then the corresponding data connection can be routed on the other path. This method is denoted `Fixed-Alternate Routing`, see e.g. Birman and Kershenbaum [13]. When the number of shortest paths for each data connection is limited to $k, k > 0$, then the `Fixed-Alternate Routing` may be referred to as the k-shortest path method, see Banerjee et al. [6] or for a general k-shortest path algorithm, see Eppstein [23]. As there are more paths to choose from, the risk of being unable to assign wavelengths to certain data connection is generally lowered. The wavelength assignment may, though, become harder to solve because of the potential many combinations of paths to choose from.

Adaptive Routing

`Adaptive Routing` is yet another routing method. It consists of finding paths with respect to previously chosen paths. Given is a network with an edge for each pair of fiber and wavelength in the network. An edge has weight 1 when unused, and ∞ when used. The path of a data connection request is found as the shortest path with respect to edge weights. The weights of edges used by this path are set to ∞, and the next data connection request can now be considered. If some nodes have wavelength converters, then an appropriate cost for converting wavelengths can be introduced. See Zang et al. for more details [71].

Least Congested Path Routing

Another `Adaptive Routing` method is the `Least Congested Path Routing`, see Chan and Yum [14]. A sequence of paths is preselected, and once a data connection request arrives, the `Least Congested Path Routing` is chosen. Least congestion is measured on the number of available wavelengths on each edge; the congestion of a path is determined by the used edge with fewest available bandwidths.

Shortest Path Adaptive Routing

Yet another method is to use the `Shortest Path Adaptive Routing`, which is an extension of the methods described above. If several paths with same cost exist, then the least congested of those paths is chosen. To determine the least congested path, all edges on all paths for a data connection must be investigated. This can be time consuming, thus Li and Somani [52] have suggested to only check the first k edges.

Routing with Reduction of Wavelength Continuity Conflicts

Recall, that when a node does not have a wavelength converter attached, then we say, that a path must have wavelength continuity in this node, i.e., a path cannot change wavelength. When several paths compete for the same wavelength on an edge and the start node of that edge does not have a wavelength converter, then we have a *wavelength continuity conflict*. When finding paths for data connection requests, we obviously wish to reduce the number of wavelength continuity conflicts. For this, Koster and Scheffel [43] present a mathematical formulation for finding a lower bound on the number of connections which cannot be routed without wavelength conversion. The bound is based on the number of incident fibers and the number of wavelength per fiber as shown by Koster in [42]. The mathematical formulation is a variant of the linear `Multicommodity Flow Problem` (`MCFP`), which is polynomial solvable. Koster and Scheffel solve the formulation using column generation. If the routing problem is solvable, then Koster and Scheffel show that it is possible to assign a wavelength to all selected paths.

Ant Colony Routing

The `Ant Colony Routing` approach is a metaheuristic. Ants are capable of finding shortest paths when working together: assume that two ants have encountered some food, and that two different paths back to the nest exist. Each ant takes its own path; on its way it lays pheromone for signaling. The path of the first ant to arrive at the nest is the shorter of the two paths, and it is the only path with pheromone all the way to the nest at this moment. Once the first ant has returned to the nest, a number of ants are sent out towards the food, all leaving pheromone on their way. The strength of the pheromone determines which path the ants choose. Thus all ants will eventually choose the (shortest) path, used by the first arriving ant. The behaviour of ants has inspired the `Ant Colony Optimization` (`ACO`). When establishing several paths, a colony of ants is assigned to each path. Ants are only attracted to the pheromone from their own colony. Varela and Sinclair [69] have proposed several `ACO`s, where ants not only are attracted to pheromone of their own colony; they are also repelled by the pheromone of other colonies.

Genetic Algorithm for Routing

Banerjee et al. [6] use a `Genetic Algorithm` (`GA`) for solving the routing problem of RWA. The `GA` is a metaheuristic. Banerjee et al. seek to minimize the number of used wavelengths *and* the average delay on a network satisfying the wavelength continuity constraint. In `GA` a number of chromosomes are given; each chromosome consists of a number of genes.

The `Genetic Algorithm` of Banerjee et al. works as follows: k-shortest path is used as routing heuristic. Each gene in a chromosome represents a path. The cost of each chromosome equals the total cost of the used edges. The cost of an edge depends on the number of paths in the chromosome using that edge. If the edge is only used once, then the cost is relatively low. If the edge is used by several (different) data connection requests, then the cost is very large. Banerjee et al. seek to minimize the cost of selected chromosomes. They thus seek to limit blocking occurring from several paths using the same edge.

Linear programming

The routing problem is formulated mathematically by Zang et al. [71]. The objective is to minimize the maximal number of paths on an edge. Zang et al. argue that this is an `Integer Multicommodity Flow Problem` (`IMCFP`), where a data connection is represented by a commodity with one amount of flow. The `IMCFP` is $\mathcal{NP}$-hard, see e.g. Barnhart et al. [9], thus Zang et al. suggest reducing the search space by only considering a subset of possible paths. Furthermore, they suggest using random rounding when solving an LP-relaxed formulation.

3.1.1 Performance of routing methods

So far the performance of the presented methods has not been discussed. In the literature, the test instances and the objective function vary. An often used objective is *blocking probability*, which gives the probability of a data connection request to be blocked, because there is no available wavelength on its path. In this section, we attempt to give an overview of problem instances and results. Despite the difference of used test instances and of objectives, we seek to provide an insight into the overall performance of the proposed methods.

Birman and Kershenbaum [13] compare `Fixed Routing` and `Fixed-Alternate Routing` on a single-hop mesh network with 6 nodes, 9 edges, a data connection request for each pair of nodes, and 24 wavelengths per edge. The objective is blocking probability, and their results show, that `Fixed-Alternate Routing` performs better than `Fixed Routing`. No running times are reported.

Chan and Yum [14] test the `Least Congested Path Routing` heuristic on a fully-connected network with seven nodes, and with thirty wavelengths per edge. The computational evaluation is based on changing parameters in the algorithm and in the network. The objective is blocking probability, and they test the effect of having different network topologies and different settings for wavelength converters rather than comparing with an existing routing heuristic. Running times are not mentioned.

Furthermore, Li and Somani [52] have compared the `Least Congested Path Routing` heuristic with the shortest path algorithm on a 4×4 mesh-torus network and on the NFS network with 14 nodes and 21 edges. Their objective is blocking probability, and the least congest path routing heuristic has best performance. Running times are not reported.

Koster and Scheffel [43] test the `Routing with Reduction of Wavelength Continuity Conflicts` on a German, European and US network, where the number of eligible paths between two nodes is limited to 100. The number of wavelengths per fiber is set to 40 and 80 in different test runs. In their test, they incorporate the routing scheme in a mathematical formulation for the RWA. They solve the formulation by using `CPLEX`, version 9.1 and compare different settings of the algorithm instead of comparing with other heuristics. A fixed time limit is set to 10000 seconds; apart from that, time usage is not mentioned.

Varela and Sinclair [69] test their variants of the `Ant Colony Routing` approach on three networks. The first has 4 nodes and 20 wavelengths. The second has 9 nodes and 98 wavelengths. The last network has 15 nodes and 269 wavelengths. The objective is to minimize the number of required wavelengths and running times are not considered. The `Ant Colony Routing` approach is compared to a heuristic with `Fixed-Alternate Routing` like method and with `First Fit Assignment`, and the latter has slightly better performance than the metaheuristic.

Banerjee et al. [6] test the `Genetic Algorithm` for routing on a number of networks. The considered simulation networks are real life networks: the 20 node ARPA network, 18 node European optical network, 22 node UK network and 14 node NSF network. Several sets of data connections are tested: 20, 40, 60, 80, and 100 data connections. The objective is to minimize the number of required wavelengths. For less than 80 data connections, the `First Fit Assignment` heuristic and the `Genetic Algorithm` perform equally well. For 80 or more data connections, the `Genetic Algorithm` finds better solutions, i.e., solutions requiring fewer wavelengths. Running times are not reported.

3.1.2 Theoretical running times

We now report theoretical running times for the presented constructive heuristics for the routing problem. To calculate the times, some notation must be introduced. Given a network, G, let N be the number of nodes, and E the number of edges. The number of wavelengths is denoted W, and let k be taken from the k-shortest path algorithm. Running times for the heuristics for the routing problem are calculated as the time it takes to find path(s) for each data connection.

The `Fixed Routing` and the `Adaptive Routing` heuristics are shortest path problems, which can be solved in $\mathcal{O}(E + V \log V)$ time using Dijkstra's algorithm, see e.g. Cormen et al. [18].

The `Fixed-Alternate Routing` problem finds the k shortest paths, which can be found in $\mathcal{O}(E + V \log v + k)$ time, see e.g. the work of Eppstein [23].

In the literature, only very large running times are given for the `Least Congested Path Routing` and the `Shortest Path Adaptive Routing` problems, see [52]. Here, we thus present a somewhat naïve algorithm for the `Least Congested Path Routing` with lower running time. The problem consists of finding a path, where the smallest number of available wavelengths on any used edge is maximized. Now, given a network and a data connection, delete the edge with fewest available wavelengths and set all other edge weights to 0. Solve the shortest path problem using Dijkstra's algorithm. If the problem is solvable, then delete the edge with second fewest available wavelengths. Resolve the problem. Continue until the problem is no longer solvable. Then we know that we have to use the just deleted edge, which has fewer available wavelengths than the remaining edges. This very straight-forward method has running time $\mathcal{O}(E(E + V \log V))$, which can surely be improved. The running time of the `Shortest Path Adaptive Routing` problem is the same, as the problem is a mix of the `Fixed-Alternate Routing` and the `Least Congested Path Routing`.

The `Routing with Reduction of Wavelength Continuity Conflicts` method presented by Koster [42], is a variant of the polynomially solvable linear `MCFP`. Koster solves the problem using column generation. To the best of our knowledge, no constructive solution method for the linear `MCFP` exists. In the literature, large instances of the linear `MCFP` are typically solved using Lagrangian methods, partition methods, decomposition techniques, dual ascent algorithms, bundle methods, interior point methods, etc., see Awerbuch and Leighton [5] and

Kennington [41] for surveys of the problem, and Larsson and Yuan [49] for a review of solution techniques. Small instances are typically solved using the Simplex algorithm. An exact running time for the `Routing with Reduction of Wavelength Continuity Conflicts` is thus difficult to calculate; instead, we simply state that the problem is polynomial.

3.2 Wavelength Assignment

When paths are found for all data connections, then wavelengths must be assigned to each path. Wavelength assignment is an $\mathcal{NP}$-hard problem.

In this section, three different types of approaches are described: theoretical results on the number of needed wavelengths, an exact graph coloring approach, and finally a number of heuristics and metaheuristics for the wavelength assignment problem.

The theoretical results on the number of wavelengths needed often depend on the network topology. The research area is quite vast, so we only give a short overview here.

Solving the wavelength assignment problem to optimality is typically done through a graph coloring problem. Much research has been conducted on the graph coloring problem; here we only show the transformation from the wavelength assignment problem to the graph coloring problem, and then give references for further information on solution methods.

For the heuristics, we assume that the number of available wavelengths is fixed. The wavelength assignment problem thus consists of finding a feasible solution, rather than finding a feasible solution which minimizes the number of used wavelength. The heuristics may be used for both the static and the dynamic wavelength assignment problem. Each path is treated separately without paying attention to the wavelength assignments of other paths. Some of the heuristics work on both the single-fiber and the multi-fiber network.

Theoretical Results on the Number of Needed Wavelengths

Once routing is done, wavelengths are to be assigned to the data connections. Much research is done on theoretical bounds on the number of required wavelengths. Especially, lower bounds on the number of wavelengths are given, i.e., given a set of paths then at least a certain number of wavelengths are needed for assignment of those paths. The bounds can be used to quickly determine whether or not all data connections can be assigned wavelengths. The bounds, however, often depend on the chosen routing algorithm. Work has also been performed on upper bounds; these bounds can be used to ensure feasibility, i.e., given a routing and given a number of available wavelengths larger than the upper bound, then a feasible wavelength assignment is guaranteed.

The research area of bounds on wavelengths is vast, as much work is done on specific network topologies. In the following, a selection of results from the research area is presented.

First, Aggarwal et al. [1] present previous work on lower and upper bounds in wavelength assignment, and then Aggarwal et al. improve the upper bounds. Their bounds apply for specific instances of the RWA. Two variants of the dynamic RWA is considered: (1) all data connections can always be rerouted, and (2) no data connection can ever be rerouted. Furthermore, they make the assumption that `Fixed Routing` is used. The network topologies include star networks, having no converters or having converters at all nodes. Aggarwal et al. find upper bounds close to previously found lower bounds. For more details on their bounds and on earlier found bounds, see the overview of previous work presented by Aggarwal et al.

Raghavan et al. [61] present heuristic algorithms for the static RWA on certain network topologies. The algorithms have bounds on the number of wavelengths needed. The network topologies include sparse, bounded degree rings, trees, and meshes, all with constraints on how to forward data in a node. Furthermore, Raghavan et al. discuss using their algorithms for the dynamic RWA.

When calculating bounds, Barry and Humblet [10] allow blocking, that is, some data connections may be blocked instead of the telecommunication provider upgrading the network. The same applies for Ramaswami and Sivarajan [63], and Yates et al. [70].

Gersel et al. [26] present algorithms with known worst upper bounds on the number of wavelengths needed for the RWA with no blocking. Their work is on certain undirected network topologies: line, ring networks and trees, all with no wavelength converters. Furthermore, they extend their results when wavelength conversion is allowed. Their algorithms are greedy heuristics, where they have added lower and upper bounds on the number of wavelengths to avoid blocking.

Koster [42] solves the wavelength assignment problem by transforming it into an *edge coloring* problem. This transformation is only possible, when no path uses more than two edges, which is the case in a star network. Koster gives lower bounds on the number of wavelengths to assign. In the case that all paths must be assigned wavelengths, Koster gives a lower bound on the number of needed wavelength converters.

The results in this section suffer from only working on specific instances of the RWA and of the underlying network. Furthermore, some results do not take blocking into account. Much more work has been conducted to finding bounds for the number of wavelengths, but to the best of out knowledge, all this work suffers from constraints set on the network topology, on the paths to assign wavelengths, etc.

Graph Coloring

The wavelength assignment problem can be solved using graph coloring methods, see e.g. Zang et al. [71]. Garey et al. [25] prove that the graph coloring problem is $\mathcal{NP}$-hard. For more general information on the graph coloring problem, see Jensen and Toft [36]. An auxiliary graph G' is constructed such that each path in the routing solution is represented by a node in G', and such that two nodes are connected in G' if the corresponding paths travel on the same fiber in the routing solution. Now, the graph coloring problem is to assign colors to all nodes in G' such that two adjacent nodes do not share the same color. This corresponds to assigning wavelength to paths such that two paths using the same fiber do not share wavelength. In graph coloring, the *chromatic number* denotes the minimum number of needed colors. Minimizing the chromatic number thus corresponds to minimizing the number of needed wavelengths. The graph coloring problem solves the wavelength assignment problem to optimality.

For information on exact, heuristical and approximate graph coloring algorithms, we refer to Pardalos et al. [58] and to the bibliography maintained by Chiarandini [15].

Random Assignment Heuristic

The `Random Assignment` algorithm consists of assigning a random available wavelength to each path. If `Fixed Routing` is used, then the `Random Assignment` algorithm is straightforward. If `Fixed-Alternate Routing` or another routing protocol is used, where each data connection request can choose from several paths, then `Random Assignment` chooses a path, which can be assigned a wavelength. If more than one path can be assigned a wavelength, then the algorithm randomly selects one of these.

The `Random Assignment` is used by e.g. Subramaniam and Barry [67].

First Fit Heuristic

This `First Fit Assignment` method consists of assigning the first available wavelength to the current path. How the first available wavelength is defined is not that relevant, as long as the order of wavelengths is predefined. The `First Fit Assignment` heuristic is widely used, see e.g. the work of Kovacevic and Acampora [47].

Least Used Heuristic

The `Least Used Assignment` heuristic selects the wavelength that is least used so far. The idea is to balance the load among all wavelengths. This approach, however, causes trouble for longer paths, as different wavelengths are all used throughout the network. Hence, the approach eventually only assigns wavelengths to short paths. For more details, see Mokhtar and Azizoglu [54] or Zang et al. [71].

Most Used Heuristic

The `Most Used Assignment` approach is the opposite of the `Least Used Assignment` heuristic. Instead of selecting the least used wavelength, this heuristic chooses the wavelength which is most used in the network. The `Most Used Assignment` heuristic is described in details by Mokhtar and Azizoglu [54] and Zang et al. [71].

Exhaustive Search Heuristic

The `Exhaustive Search Assignment` algorithm works on top of `Fixed-Alternate Routing` or another routing scheme generating several paths per data connection request. The wavelength assignment heuristic checks all available wavelengths and chooses the one, which gives the shortest path. Mokhtar and Azizoglu [54] argue that the method has quite high complexity as it needs to check all wavelengths on all paths.

Minimum Product Heuristic

The `Minimum Product Assignment` approach consists of minimizing the number of fibers used in a multi-fiber network and is introduced by Jeong and Ayanoglu [37]. Let D_{ij} denote the number of assigned fibers on edge i and for wavelength j. Then this heuristic calculates $\Pi_i D_{ij}$ for all wavelengths j.

Least Loaded Heuristic

The `Least Loaded Assignment` approach is also designed for a multi-fiber network. Given a path, the heuristic finds the wavelength, whose smallest availability is larger than that for all other wavelengths. Let M_i be the number of fibers on edge i, and let D_{ij} be the number of assigned fibers on edge i for wavelength j. Then the `Least Loaded Assignment` approach selects the wavelength j with $\max_j \min_i(M_i - D_{ij})$. For more details, see Zang et al. [71] and Karasan and Ayanoglu [40].

Maximum Sum Heuristic

Subramaniam and Barry [11, 67] present a `Maximum Sum Assignment` algorithm for assigning wavelengths. Given is a network, where paths are preselected. Now, when a new data connection request arrives and a path is found, the heuristic of Subramaniam and Barry seeks to find a wavelength, where after assignment the remaining capacity is as large as possible. Subramaniam and Barry designed the algorithm for multi-fiber network, but it also applies for single-fiber networks.

Relative Capacity Loss Heuristic

The `Relative Capacity Loss Assignment` heuristic is introduced by Zhang and Qiao [72], and it is a variant of the `Maximum Sum Assignment` approach. The latter selects the wavelength, which minimizes the capacity loss (or maximizes the remaining capacity) on all edges. The `Relative Capacity Loss Assignment` chooses the wavelength which minimizes the relative capacity loss, i.e., the capacity loss divided with the available capacity.

Distributed relative capacity loss heuristic

Zang et al. [71] propose the `Distributed Relative Capacity Loss Assignment` heuristic for assigning wavelength. The algorithm is a variant of the `Relative Capacity Loss Assignment` heuristic. It reduces complexity of the former heuristic by generating a look-up table, such that the relative loss capacity of wavelengths is readily available. The look-up table is build by investigating the network and by exchanging information between nodes in a manner similar to that of the Bellman-Ford shortest path algorithm, see Cormen et al. [18] for the Bellman-Ford algorithm.

Wavelength Reservation Heuristic

As the name of the `Wavelength Reservation Assignment` heuristic indicates, this method reserves wavelengths for certain data connections. An example is, that a wavelength λ is reserved for all data going from a node a to node c. If several paths have a and c as intermediate nodes, then they compete for the reserved wavelength, λ. Note, that another wavelength assignment method must be used to determine which path to select for the current data connection, and which wavelength to reserve. Birman and Kershenbaum [13] introduce the wavelength heuristic approach for multi-hop connections, and they show that it reduces the blocking for multi-hop connections, but it also increases the blocking for single-hop connections.

Protecting Threshold Heuristic

Birman and Kershenbaum [13] introduce the `Protecting Threshold Assignment` approach, which consists of only selecting a wavelength when the number of idle wavelengths on the edge is above a certain threshold. Note, that another wavelength assignment must be used to determine which path to select for the current data connection and which wavelength to assign to the path. Birman and Kershenbaum have developed the heuristic for single-hop data connections.

Genetic algorithm

`Genetic Algorithms` (`GA`) try to simulate evolution of genotypes and natural selection, see e.g. Goldberg [28]. Hyytiä and Virtamo [30] suggest a `GA` for solving the wavelength assignment problem as a graph coloring problem. Two chromosomes are given, each representing a solution to the graph coloring problem. A new chromosome is generated from the two previous chromosomes; the reuse of a chromosome depends on the quality of the corresponding solution (which is the number of used wavelengths). The new chromosome represents a solution to the wavelength assignment problem.

Simulated annealing

`Simulated Annealing` (`SA`) is based on resolving the problem and accepting a new and better solution with some probability. This probability depends on a *temperature* parameter, which decreases with time. Hence, the name simulated *annealing*. For more details, see van Laarhoven and Aarts [68]. Hyytiä and Virtamo [30] present a `SA` approach used on the wavelength assignment problem. The problem is considered as a graph coloring problem, and the `SA` consists of assigning different colors to nodes, calculating the objective cost, i.e., the number of used wavelength, and then accepting the new solution with some probability.

Tabu search

Finally, `Tabu Search` (`TS`) is based on a random search approach where certain moves are forbidden or *tabu*, see e.g. Glover and Laguna [27]. Hyytiä and Virtamo [30] suggest solving the wavelength assignment problem represented by a graph coloring problem, by using `Tabu Search`. The objective is to maximize the number of established connections rather than to minimize the number of used wavelengths.

Bin Packing Heuristic

The RWA on a network with no wavelength converters can be solved by applying the bin packing problem. For more information on the bin packing problem, see Pisinger and Sigurd [60]. Skorin-Kapov [65] represents the RWA as a bin packing problem by letting paths be items, and by letting copies of the network be bins. Each bin represents a wavelength, and each bin has capacity equal to the number of edges in the network. Two items cannot be packed in the same bin if the corresponding paths use the same edge. Now, the bin packing problem is to pack items into as few bins as possible. This corresponds to minimizing the number of assigned wavelengths.

3.2.1 Performance of wavelength assignment methods

Again, the performance of the presented methods has not been discussed, because in the literature the test instances and the objective function vary. In this section, we give an overview of the performance of the wavelength assignment methods, including a description of the evaluated problem instances and the corresponding evaluation results.

Kovacevic and Acampora [47] compare the `First Fit Assignment` heuristic for wavelength assignment with the `Random Assignment` approach. The test instance is a 11×11 mesh network with 5 wavelengths per edge, and with varying network load. The objective is blocking probability and the results show that the `First Fit Assignment` heuristic generally gives better results than the `Random Assignment`. Running times are not mentioned.

Mokhtar and Azizoglu [54] compare the `Exhaustive Search Assignment` with the `Most Used Assignment` algorithm, the `First Fit Assignment`, and `Random Assignment`. The test instances are two networks: the ARPA-2 network with 21 nodes, 26 edges and 4 or 8 wavelengths, and a randomly generated topology with 15 nodes and 32 edges. Traffic arrives according to the Poisson process. The objective is blocking probability. The `Most Used Assignment`, `Random Assignment` and `Least Used Assignment` heuristics are tested on both networks. The `Most Used Assignment` heuristic performs best, followed by `Random Assignment`. Then the `Exhaustive Search Assignment` algorithm is compared to the `Most Used Assignment`, and the `Exhaustive Search Assignment` algorithm gives slightly better results, but Mokhtar and Azizoglu note, that the increased complexity of the `Exhaustive Search Assignment` overshadows the better results. `First Fit Assignment` is compared with the `Most Used Assignment` heuristic, and `First Fit Assignment` performs almost equally well to the `Most Used Assignment` method. Time usage is not given, but theoretical complexities are computed for the heuristics.

Karasan and Ayanoglu [40] implement the `Least Loaded Assignment` heuristic. They test it on a 30-node mesh network where traffic is distributed uniformly. The network reflects the geographical location of major cities in the US. Connection requests arrive according to the Poisson process. The network is either single-fiber or multi-fiber, each fiber having 8 wavelengths. The objective is blocking probability. Results show, that the `Least Loaded Assignment` heuristic performs better than the `Most Used Assignment` approach.

Subramaniam and Barry [67] test the `Random Assignment`, `First Fit Assignment`, `Least Loaded Assignment`, `Most Used Assignment`, `Minimum Product Assignment` and the `Maximum Sum Assignment` heuristics. The instances have uniform Poisson traffic, and are either a 20 node ring network with 1 or 10 fibers per edge, or a 5×5 bidirectional mesh-network with 1 or 3 fibers per edge. Subramaniam and Barry use blocking probability as objective. Running times are not mentioned. According to Subramaniam and Barry the `Minimum Product Assignment` heuristic performs slightly better than the `Most Used Assignment` heuristic with respect to blocking probability. Then follows the `First Fit Assignment`, `Least Loaded Assignment`, `Maximum Sum Assignment` and finally the `Random Assignment` heuristics.

Zhang and Qiao [72] test the `First Fit Assignment`, the `Maximum Sum Assignment` approach and the `Relative Capacity Loss Assignment` heuristic on a simulation of the NFS network and on a 4×4 torus network. They use blocking probabilities to calculate their objective function value. The `Relative Capacity Loss Assignment` method has best performance.

Zang et al. [71] compare a number of heuristics for wavelength assignment: `Random Assignment`, `First Fit Assignment`, `Least Used Assignment`, `Most Used Assignment`, `Minimum Product Assignment`, `Least Loaded Assignment`, `Maximum Sum Assignment`, and `Relative Capacity Loss Assignment`. A network consisting of six nodes is used for testing, where the number of wavelengths and fibers vary. The objective is blocking probabilities, and practical running times are not mentioned. In a single fiber network, the `Most Used Assignment` heuristic performs well, along with the `Maximum Sum Assignment` and `Relative Capacity Loss Assignment` approaches when the load is low. When the load is high, then all heuristics have similar performance. In a multi-fiber network, the `Most Used Assignment`, `Minimum Product Assignment` and `Relative Capacity Loss Assignment` methods have best performance, while the `Least Loaded Assignment` and `Maximum Sum Assign-

`ment` heuristics work best with high load. Zang et al. conclude, however, that the difference between the performances of all heuristics is quite insignificant.

Birman and Kershenbaum [13] compare the `Wavelength Reservation Assignment` and the `Protecting Threshold Assignment` heuristics on a single-hop mesh networks with 6 nodes, 9 edges, a data connection request for each pair of nodes, and 24 wavelengths per edge. The objective is blocking probability, and the results show, that the `Protecting Threshold Assignment` algorithm tends to give better results than the `Wavelength Reservation Assignment` approach. No running times are given.

The metaheuristics suggested by Hyytiä and Virtamo [30] include a `Genetic Algorithm`, `Simulated Annealing` and `Tabu Search`. The methods are compared with each other and with a `First Fit Assignment` heuristic, on randomly generated instances not described any further. The results show, that the greedy heuristic has significantly better running time. The `Genetic Algorithm` has better running time than the `Simulated Annealing`, which is faster than the `Tabu Search`. The methods are also compared with respect to the number of generated wavelengths. Here, the `Tabu Search` has best performance, followed by the `Genetic Algorithm`, the `First Fit Assignment`, and finally the `Simulated Annealing`.

Skorin-Kapov [65] tests the `Bin Packing Heuristic` on a series of random 100-node networks with average degrees of 3, 4, and 5. Random sets of data connections requests were created for each test network with a fixed probability of there being a data connection request between two nodes. The number of requests varies from 2054 to 9900. The objective is to minimize the number of required wavelengths along with the length, in hops, of data connections. The results show that the heuristics find optimal or near-optimal solutions. Running times are mentioned to be low in general: solving an instance with 100 nodes and 9900 data connection requests takes less than 8 minutes on a P4 2.8 GHz processor.

Zang et al. [71] argue that the routing algorithm has larger influence on the amount of blocking probability, than the wavelength assignment algorithm. They base this on the performed tests, where algorithms using `Adaptive Routing` generally gives significantly better results than algorithms using `Fixed Routing` - no matter which wavelength assignment algorithm is used. Zang et al., however, do not take running times into account so even if more complicated routing algorithms give better solutions, one could fear that the algorithms may also have larger time usage.

3.2.2 Theoretical running times

We now report theoretical running times for the presented constructive heuristics for the wavelength assignment. Recall the notation: given a network, G, let N be the number of nodes, and E the number of edges. The number of wavelengths is denoted W, and let k be taken from the k-shortest path algorithm. Running times for the wavelength assignment heuristics are calculated as the time it takes to assign a wavelength to a single path.

The `Random Assignment` selects a random wavelength. In the case of no wavelength converters, the heuristic investigates all edges on the path to see if the wavelength is available; if not, it repeats the process with another randomly picked wavelength. In the case of wavelength converters, the heuristic investigates if the wavelength is available on each edge, and if not, it selects another wavelength and check again. The running time is $\mathcal{O}(WE)$. The `First Fit Assignment` only differs in how to pick the wavelength, and it thus has the same running time.

The `Least Used Assignment` and `Most Used Assignment` heuristics run through all used edges and calculate how much each wavelength is used. The wavelengths are sorted according to usage, and paths are assigned wavelengths from the sorted list in a `First Fit Assignment` manner. The running time is $\mathcal{O}(W \log W + WE)$.

The `Exhaustive Search Assignment` needs to check all available wavelengths on all k paths for the current data connection. This takes $\mathcal{O}(kWE)$ time.

The `Minimum Product Assignment` heuristic calculates the product $\Pi_i D_{ij}$ for all fibers i and for all wavelengths j. This takes $\mathcal{O}(WE)$ time. The `Least Loaded Assignment` heuristic is very similar to the `Minimum Product Assignment` method and thus has the same running time, $\mathcal{O}(WE)$.

The `Maximum Sum Assignment` heuristic investigates how much each wavelength is available on each edge of all the paths, the current data connection can choose from. Let the number of paths be bounded by k; the running time is $\mathcal{O}(kWE)$. The `Relative Capacity Loss Assignment` heuristic and the `Distributed Relative Capacity Loss Assignment` heuristic work in a similar manner and thus have the same running time, $\mathcal{O}(kWE)$.

Finally, the `Wavelength Reservation Assignment` heuristic and the `Protecting Threshold Assignment` heuristic are used on top of other wavelength assignment algorithms. Thus, their running times depend on the other heuristic: the `Wavelength Reservation Assignment` and `Protecting Threshold Assignment` methods themselves have constant running time, $\mathcal{O}(1)$.

4 Overall Methods for Solving the RWA Problem

In this section, important results for solving the RWA as one problem are presented. Instead of splitting the RWA into two subproblems, the following methods approach the entire RWA. Methods include both metaheuristics and exact formulations.

4.1 Metaheuristics

In this section, metaheuristics for solving the RWA are presented. The metaheuristics proposed in the literature are `Genetic Algorithm` (`GA`) and `Ant Colony Optimization` algorithms (`ACO`).

Ant Colony Optimization Algorithm

Arteta et al. [4] use a `multi-objective` (`MO`) `ACO` metaheuristic for solving the RWA. `ACO` defines a method of investigating the neighbourhood of a current solution. The `MO` consists of optimizing the hop count and the number of wavelength conversions. In the `ACO` this means, that the pheromone matrix, i.e., the probabilities defining which pheromone track an ant chooses, depends on the path's hop count and on the number of wavelength conversions in the path.

Arteta et al. have implemented several `MOACO`s: for more details on each `MOACO`, see the corresponding reference. The `multiple objective ant Q algorithm` (`MOAQ`) of Mariano and Morales [53] maintains a colony per objective. The `bicriterion ant` (`BIANT`) of Iredi et al. [33] uses a probability matrix per objective, and hence also a colony per objective. `Pareto ant colony optimization` (`PACO`) of Doerner et al. [20] has several pheromone matrices for each objective. The `Multi-objective ant colony system` (`MOACS`) by Schaerer and Barán [8] uses several heuristics when calculating entries in the probabily matrix. The `multi-objective max-min ant system` (`M3AS`) by Pinto and Barán [59] has a global pheromone matrix. `COMPETants` (`COMP`) by Doerner et al. [21] uses several heuristics, pheromone matrices and the colony sizes vary. `Multi-objective omicron ACO` (MOA) by Gardel el al. [24] uses a specific updating rule for the pheromone matrices, and finally `multi-objective ant system` (`MAS`) by Paciello et al. [57] has a slightly different order of updating the pheromone matrices.

Genetic Algorithms

Sinclair [64] solves the RWA through a `Genetic Algorithm` (`GA`). Instead of using the classical mutation and crossover operations in `GA`, Sinclair uses heuristics to generate new solutions. The heuristics are: k-shortest path routing with `First Fit Assignment`, rerouting and reassignment of wavelength of a subset of connections, rerouting a path with high wavelength in order to reach the lowest possible wavelength, and shifting the path with the highest wavelength to having a lower wavelength such that all paths blocking the new low wavelength must be rerouted.

Ali et al. [3] solve a variant of the RWA problem using a `Genetic Algorithm`. The variant consists of taking power into account, i.e., they wish to preserve proper power levels on all paths. They use a k-shortest path method to generate routes, where power loss is taken into account when measuring the length of a path.

4.2 Linear Programming

This section presents methods from the literature for finding LP bounds for the RWA. Several of the methods presented in the following may be integer or mixed integer programs, but the suggested solution methods all work on LP relaxed formulations.

Ramaswami and Sivarajan [63] present an `Integer Programming` (`ILP`) formulation for the static RWA with no wavelength conversion, and where the objective is to maximize the number of established data connections. They note that their model is a variant of the `MCFP`. Given the data connections and corresponding paths, Ramawami and Sivarajan solve the problem using rounding algorithms. Data connections and paths are generated randomly.

An `ILP` formulation of the static RWA is presented by Zang et al. [71]. Wavelength conversion is not allowed, and the objective is to minimize the maximal edge flow. It is noted, that the model is a variant of the `MCFP`. Zang et al. also present an overview of a model for the static RWA with wavelength conversion, which again is a variant of the `MCFP`.

Banerjee and Mukherjee [7] present an `ILP` for the RWA, where the objective is to minimize the hop distance. The network allows wavelength conversion. They, however, solve the problem heuristically. Banerjee and Mukherjee argue that their model can be used to design a balanced network with high utilization of transceivers and wavelengths. Furthermore, it is noted, that the model of Banerhee and Mukherjee is a variant of the `MCFP`, where each commodity represent a data connection.

Ozdaglar and Bertsekas [56] work on an `ILP` formulation of the quasi-static RWA. They define quasi-static RWA to be the problem, where several data connection requests first are to be handled, and then later more data connection requests may arrive. The formulation is a variant of the `MCFP`. Ozdaglar and Bertsekas relax the `ILP` and show that the relaxed formulation yields integer solutions for several network topologies including line and ring networks, with wavelength converters at either all or no nodes.

Jaumard et al. [34] present a number of different ILP formulations for the RWA in WDM optical networks, using a unified notation. The variants of the RWA include instances with symmetric and with asymmetric traffic. Jaumard et al. show edge- and path-based formulations as well as models from the literature. Formulations for the RWA with asymmetric traffic are shown to have the same objective, though the number of constraints and variables differ.

4.3 Integer Programming

This section presents exact solution methods for the RWA. The methods are all based on Dantzig-Wolfe decomposing the RWA, see [19]. The resulting formulations are solved to optimality using branch-and-price, where the master and subproblems vary according to the used Dantzig-Wolfe decomposition.

If wavelengths may be changed in every node, the RWA problem can be reduced to the `Integer Multicommodity Flow Problem` (`IMCFP`), see Beauquier et al. [12]. The `IMCFP` consists of sending an amount of flow between several sources and targets with respect to edge capacities, see Ahuja at el. [2] for more details. When wavelengths can be converted at all nodes, then the wavelength limitation can be described as edge capacities: each edge can carry at most k different wavelengths, for some integer $k > 0$. Now, we need to send 1 amount of flow between all data connection terminals without violating edge capacities. This corresponds to the integer `MCFP`. The integer `MCFP` is a well-studied problem with many solution approaches. An example is the branch-and-bound algorithm by Barnhart et al. [9].

Another `ILP` formulation for the RWA is of Lee et al. [51] which is based on finding a set of paths with the same wavelength for a subset of data connection. The formulation maximizes the number of established data connections subject to the RWA constraints. Lee et al. propose a column generation for the formulation, where the subproblem is to find a set of paths with the same wavelength for some data connections. To find an optimal solution Lee et al. present a branch-and-price algorithm.

Jaumard et al. [35] analyze column generation formulations for the RWA from the literature and present a new formulation. First a straight forward path formulation of the RWA is presented, where a path consists of both the visited edges and the used wavelengths. It is argued that the formulation yields symmetry problems with respect to the used wavelengths. Then Jaumard et al. review the formulation of Ramaswami and Sivarajan [63] where wavelength assignment and path variables are kept separately. Jaumard et al. propose a column generation method for generating paths for the formulation, however, the method has some drawbacks: the size of the subproblem depends on the number of paths for a data connection which may be exponential and the column generation technique solves the LP relaxed formulation and does thus not return an optimal solution to the original problem. Jaumard et al. present the formulation of Lee et al. based on finding a set of paths with the same wavelength for a subset of data connections. Jaumard et al. suggest solving the subproblem as a multicommodity linear flow problem. Based on the formulations of Ramaswami and Sivarajan [63] and Lee et al. [51], Jaumard et al. propose a new mathematical model where each column consists of a set of paths for a subset of data connections and where wavelengths are assigned in the master problem. A branch-and-price algorithm is presented where the subproblem corresponds to that of the formulation of Lee et al. and the branching strategy cuts on the number of used wavelengths are added to the master problem. Jaumard et al. have implemented and tested the column generation formulation of Lee et al. and of their own model.

4.4 Comparison of overall solution methods

Once again, the test instances and the objective function vary in the literature. An overview of tested instances and corresponding results for the overall solution methods is presented in this section.

Arteta et al. [4] test their `MOACO` metaheuristics for solving the RWA on the Japanese NTT network topology. The network has 55 nodes and 144 edges. The algorithms were run 10 times, each time of at most 100 iterations. The objective is to minimize the amount of wavelength conversion and the hop length, along with pareto front and error. Running times are not considered. Using this objective, the `MOACO`s outperform simpler, greedy heuristics.

Sinclair [64] solves the RWA through a `Genetic Algorithm`. Five test networks are generated, each with 15 nodes, and with 34 to 39 edges. The objective is to minimize the cost of used edges, and running times are not taken into account. Sinclair shows that the proposed `Genetic Algorithm` can compete with greedy heuristics.

Ali et al. [3] solve a variant of the RWA problem using a `Genetic Algorithm`. They test their algorithm on a network with 13 nodes, and the objective is to maximize the number of established data connections and in time usage. The proposed `Genetic Algorithm` outperforms a `First Fit Assignment` like heuristic with respect to the number of data connections, but it spends significantly more time.

Ramaswami and Sivarajan [63] present an `ILP`. They solve the problem using a rounding method, and they compare their bounds with a `First Fit Assignment` like heuristic. The test instances are two networks with data connection requests arriving according to a Poisson process and lasting for a duration that is exponentially distributed. The networks are a 5 node pentagon and a 20 node network representing a skeleton of ARPA, respectively. First off, Ramaswami and Sivarajan show that they reach their theoretically calculated bounds on carried traffic. They compare their rounding method for the `ILP` with the heuristic with respect to blocking probability, and their rounding method gives best results. Running times are not taken into account.

Banerjee and Mukherjee [7] present an `ILP` to derive a minimal hop distance solution in a network with wavelength converters. Two heuristics are proposed: One which attempts to find paths between the node pairs, which have more data connection requests, and which are only separated by a single hop. The other heuristic attempts to maximize the number of established data connections with respect to the number of hops between the sources and targets. Banerjee and Mukherjee test the heuristics and the `ILP` on the NFS network with a randomly generated traffic matrix. They show that the average packet hop distance for the heuristical solutions is not far from that obtained by the `ILP`. Running times are not mentioned.

Jaumard et al. [34] test the models on NSF and EON networks with asymmetrical traffic matrices of Krishnaswamy, which corresponds to 268 connections for the NSF instance and 374 for the EON. For symmetrical traffic, the former are modified such that for a pair of nodes s, d, then the selected connections are the connections from s to d, unless the number of connections from d to s is larger. This gives 191 connections for the NSF, and 270 for the EON. Formulations are compared through computational evaluation, and they show that benchmark problems from Krishnaswamy and Sivarajan [48] can be solved to optimality or with a small gap. Only bounds are compared in the computational study, hence running times are not mentioned.

Lee et al. [51] test their branch-and-price algorithm using test instances based on the SONET ring topology with 10, 15 and 20 nodes and where each node pair requires one to three data connections. Their test results show that the bounds found in the root node of the branch-and-bound tree are of good quality and optimal solutions are found for the majority of instances. An upper bound on 20000 branch-and-bound nodes is applied. Small instances are solved to optimality in seconds, while larger instances take up to 15 minutes to solve.

In the later work of Jaumard et al. [35], the column generation algorithm from Lee et al. [51] and the branch-and-price algorithm for the new formulation proposed by Jaumard et al. are implemented. They are tested and compared with solving an edge-based formulation to optimality using CPLEX. The test instances are modified NSF and EON benchmarks taken from Krishnaswamy and Sivarajan [48]. Some edges are removed from the NSF instances and extra data connections are added to the EON instances. Finally some test instances resembling a Brazilian network topology proposed by Noronha and Ribeiro [55] are used. The computational results show that the branch-and-price algorithm finds better bounds than the column generation method by Lee et al. and in less time. Furthermore, the branch-and-price algorithm is capable of finding an optimal solution for the far majority of instances and thus finds more optimal solutions than when using CPLEX on the edge-based formulation. Running times for the branch-and-price and column generation algorithms span from less than a minute for smaller instances up to days for the larger instances.

5 Conclusion

A wide variety of solution methods for the RWA have been presented. Most work in the literature is based on heuristics, more specifically on dividing the RWA into two parts: the routing problem and the wavelength assignment problem. For the main part, the heuristics apply on both the static and on the dynamic RWA.

Some work has also been concentrated on metaheuristics, both for the routing problem, the wavelength assignment problem, but also for the entire RWA. The metaheuristics work on the static RWA, as they generally seek to improve the last solution.

Less work is based on finding optimal solutions to the static RWA. In the literature it is argued, that since the RWA is $\mathcal{NP}$-hard, then finding an optimal solution is too hard. The exact solution approaches presented and tested in the literature, however, perform fairly well.

In this survey, experimental results from the literature and theoretical running times are presented. A general issue for comparing solution methods is the inconsistency in test instances and objective functions.

Running times seem to be of little interest in most experiments performed on the proposed methods. In this case, we believe that future work should focus on the `MCFP` representation of the problem. The RWA is a variant of the well-studied `MCFP`, thus algorithms for the `MCFP` need to be modified, when solving the RWA.

If running times are of interest, then the heuristics for the decomposed RWA seem to give good results fast. All greedy heuristics run in polynomial time, and their theoretical running times are generally small.

When focusing on solution qualities, then the most used objective is blocking probability. This is relevant given instances, where not all data connections can be established, and given that no general benchmark instances are used. Blocking probability tries to give a measure for the probability of the establishment of a data connection. We, however, fear that this objective is difficult to compare across the many different types and sizes of problem instances. We thus recommend the use of general instances, e.g., like the Solomon benchmark instances are used for the Vehicle Routing Problem with Time Windows [66]. General benchmark instances for the RWA could be generated randomly, or be based on known problems from general graph theory, or from some of the widely used test instance libraries available. E.g., several benchmark instances for mixed integer problems are found in the `MIPlib` (`http://miplib.zib.de/`), and a data library for fixed telecommunication network design is found in `SNDlib` (`http://sndlib.zib.de`).

As is the case in most situations dealing with $\mathcal{NP}$-hard problems, the trade-off lies between solution quality and time usage. Optimal solutions are generally only reached quickly, when the problem instances are very small. A large part of the networks, which are used for testing in the literature, are not too large.

For the static RWA problem, it may thus be beneficial to focus more on `MCFP` formulations of the RWA problem. The `MCFP` and many variants hereof are well-studied and many exact algorithms with good performance are presented in the literature. For example, the branch-and-price-and-cut algorithm for the $\mathcal{NP}$-hard `IMCFP` by Barnhart et al. [9] solves instances with up to nearly 93 commodities, 29 nodes, and 61 edges to optimality. As another example, instances for the linear `MCFP` with up to 80000 commodities, 3600 nodes and 14000 edges are solved to near-optimality by a Lagrangian algorithm presented by Larsson and Di Yuan [49].

For the dynamic RWA, the heuristics for the decomposed RWA have good performance, and we believe that any further work should concentrate on either these heuristics or on heuristics for the entire RWA.

In this survey, network design has been left out. From the perspective of a telecommunications provider, however, network design may be important, as optical networks are constantly being extended in order to reach new customers. The research area for network design is vast, thus a separate survey for this area should be consulted for further details, see e.g. Dutta and Rouskas [22], Iness [31], Jue [39] or Zymolka [74].

Solving the RWA can be used in several contexts. A solution can decide which data connections to establish. The objective may be to maximize the number of established connections, to minimize the cost of setting up connections, to minimize delays on established connections, to minimize blocking, etc. Furthermore, solution methods can be used as an analytic tool to measure performance, to measure which parts of the network is subject to most usage etc. The presented solution methods have a trade-off between solution quality and time usage. When solving the RWA, it is thus important to decide which is more important; solution quality or time usage.

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When transmitting data in an all-optical network, data connections must be established in such a way that two or more connections never share a wavelength on the same fiber. The NP-hard Routing and Wavelength Assignment (RWA) problem consists of finding paths and wavelengths for a set of data connections.

This survey introduces the RWA and gives an overview of heuristic, metaheuristic and exact solution methods from the literature. Running times for the heuristic methods are presented and computational results are discussed.

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